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Noise in electronic circuits

Jerzy Witkowski p.132a,b -4

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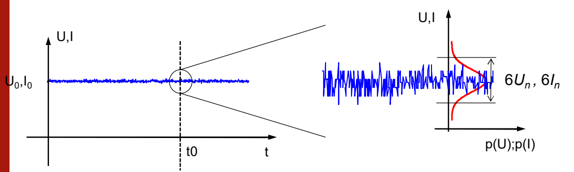
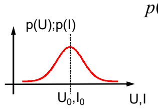
Noise in electronic circuits



$$p(U) = \frac{1}{U_n\sqrt{2\pi}} \exp\left(-\frac{(U-U_0)^2}{2U_n^2}\right)$$



$$p(I) = \frac{1}{I_n\sqrt{2\pi}} \exp\left(-\frac{(I-I_0)^2}{2I_n^2}\right)$$

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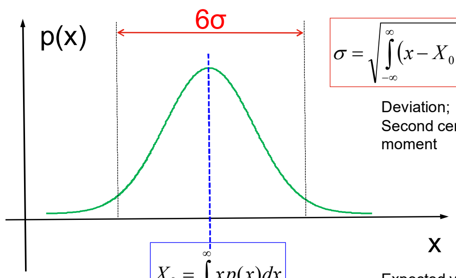
Normal distribution (Gauss distribution)



$$\sigma = \sqrt{\int_{-\infty}^{\infty} (x - X_0)^2 p(x)dx}$$

Deviation; Second central moment

$$X_0 = \int_{-\infty}^{\infty} x p(x) dx$$

Expected value; First moment

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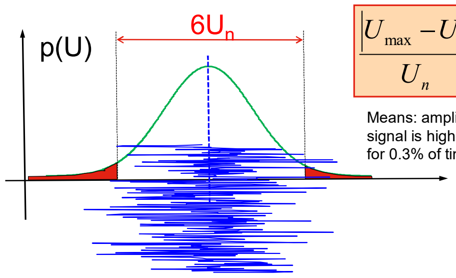
Crest factor = Max/RMS

def.: ratio of amplitude(max value) to RMS



$$\frac{\left|U_{\max} - U_o\right|}{U_n} = 3$$

Means: amplitude of signal is higher than 3Un for 0.3% of time

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RMS of current
Assumptuon: noise is stationary and ergodic process

$$P = I_{RMS}^2 R = \frac{U_{RMS}^2}{R}$$ Power

$$I_{RMS} = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T I^2(t)dt}$$ RMS

$$I_0 = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T I dt} = \int_{-\infty}^{\infty} I p(I) dI$$ Mean (average)

$$I_{n(RMS)} = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T (I - I_0)^2 dt} = \int_{-\infty}^{\infty} (I - I_0)^2 p(I) dI === I_n$$

RMS of noise component

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RMS of voltage

$$P = I_{RMS}^2 R = \frac{U_{RMS}^2}{R}$$ Power

$$U_{RMS} = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T U^2(t)dt}$$ RMS

$$U_0 = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T U dt} = \int_{-\infty}^{\infty} U p(U) dU$$ Mean (average)

$$U_{n(RMS)} = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T (U - U_0)^2 dt} = \int_{-\infty}^{\infty} (U - U_0)^2 p(U) dU == U_n$$

RMS of noise component

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Sources of noise

- Thermal noise (Johnson noise, Johnson–Nyquist noise)
- (due to fluctuations of discrete charges)
- Shot noise
- (due to electric current being the flow of discrete charges; Poisson distribution)
- Flicker noise (1/f , structural, Schottky)
- (**probably** caused by temperature fluctuations modulating the resistance)

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Other noise sources (types)

- Burst (random telegraph noise - RTN, popcorn noise, impulse noise, bi-stable noise)
  - (random trapping and release of charge carriers at thin film interfaces or at defect sites in bulk semiconductor crystal)
- Avalanche noise
  - (avalanche breakdown in reverse-biased junctions and the leakage current which can be multiplied by the avalanche phenomenon)

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Thermal noise
(Johnson,s)

- Caused by thermal fluctuation of charge
- Exist in every resistor;
- Uniform spectra density – white noise; (up to 10^{13} Hz)
- Gaussian distribution.

$$U_n = \sqrt{4kTRB}$$

$$I_n = \sqrt{\frac{4kTB}{R}}$$

k – Boltzman constant =1,38•10^{-23} [J/K]
,
T – temperature [K],
B – bandwidth [Hz],
R – resistivity.

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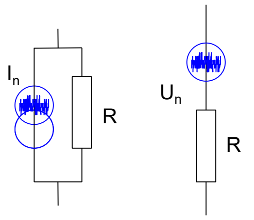
Resistor noise

$$U_n = \sqrt{4kTRB}$$

$$I_n = \sqrt{\frac{4kTB}{R}}$$



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Power & spectra density

(for resistor)

$$\frac{U_n^2}{R} = 4kTB\,[W]$$

$$I_n^2 R = 4kTB\,[W]$$

$$\frac{U_n^2}{R} \Big/ B = 4kT\left[\frac{W}{Hz}\right]$$

$$I_n^2 R \big/ B = 4kT\left[\frac{W}{Hz}\right]$$

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Practical units

(for resistors)

$$U_n = \sqrt{4kTRB} = \sqrt{4kTR}\cdot\sqrt{B}$$

$$I_n = \sqrt{\frac{4kTB}{R}} = \sqrt{\frac{4kT}{R}}\cdot\sqrt{B}$$

$$\frac{U_n}{\sqrt{B}} = \sqrt{4kTR}\quad\left[\frac{V}{\sqrt{Hz}}\right]$$

$$\frac{I_n}{\sqrt{B}} = \sqrt{\frac{4kT}{R}}\quad\left[\frac{A}{\sqrt{Hz}}\right]$$

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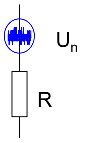
Examples



T=293 K (20° C); k=1,38•10^{-23} J/K

$$\frac{U_n}{\sqrt{BR}} = \sqrt{4kT}\left[\frac{V}{\sqrt{Hz\Omega}}\right] = 0.13\left[\frac{nV}{\sqrt{Hz\Omega}}\right] = 0.13\left[\frac{\mu V}{\sqrt{kHz\,k\Omega}}\right]$$

R	B	U_n [μV]
100 Ω	20 kHz	0,18 μV
100 Ω	2 MHz	1,8 μV
10 kΩ	20kHz	1,8 μV
75 Ω	1GHz	35 μV

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Shot noise
(Schottky noise)

- due to electric current being the flow of discrete charges; *Poisson distribution*
- Gaussian distribution – white noise.

$$I_n = \sqrt{2qI_0B}$$

Io – current,
q – elementary charge, (e=1,6•10^{-19} C)
B – bandwidth .

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Flicker - 1/f

- Excess noise (other shot & thermal)
- Power spectra is inversely proportional to frequency - Not the white noise !
- Depends of technology (materials)
- In case of biasing only (U_0>0) !,

$$U_n = K_u U_0 \sqrt{\frac{B}{f}}$$

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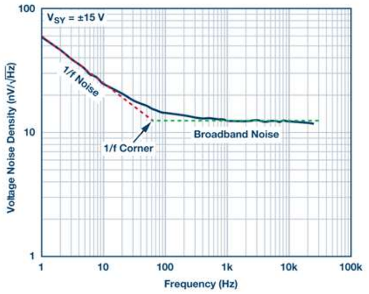
1/f power spectra (example)



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1/f
(examples for different resistors)

$$\frac{U_n}{U_0} = K_u\sqrt{\frac{B}{f}} \approx 2{,}3K_u$$

Carbon bulk	0,1 - 3 μV/V
Carbon film	0,05 - 0,3 μV/V
Metal film	0,02 – 0,2 μV/V
wire	0,01 - 0,2 μV/V

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Burst noise
(random telegraph noise - RTN, popcorn noise, impulse noise, bi-stable noise)

- Low frequency noise,
- Especially in OpAmos (not only),
- Not Gaussian !!!!

(random trapping and release of charge carriers at thin film interfaces or at defect sites in bulk semiconductor crystal)

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Avalanche noise

- avalanche breakdown in reverse-biased junctions and the leakage current which can be multiplied by the avalanche phenomenon; *Poisson distribution*.
- Electric field can accelerate electrons to high speeds, causing them to collide with other atoms and generate more electron-hole pairs (avalanche).
- White noise Gaussian distribution

$$I_n = \frac{2qI_0}{(2\pi\tau)^2}$$

τ – average collision time

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Summing up noise

$$U_n = \sqrt{U_{n1}^2 + U_{n2}^2 + \ldots + U_{nK}^2}$$

$$U_n = \sqrt{(U_{n1} + U_{n2})^2} = \sqrt{U_{n1}^2 + U_{n2}^2 + 2U_{n1}U_{n2}C}$$

Correlation coefficient
C=0

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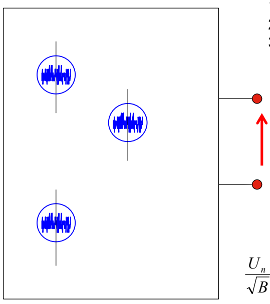
Noise calculation (not corelated noise)



1. Superposition theorem
2. Sum up squares !!!!
3. calculate the root .

$$U_n = U_{n1} + U_{n2} + \ldots + U_{nK}$$

$$U_n = \sqrt{U_{n1}^2 + U_{n2}^2 + \ldots + U_{nK}^2}$$

$$\frac{U_n}{\sqrt{B}} = \sqrt{\left(\frac{U_{n1}}{\sqrt{B}}\right)^2 + \left(\frac{U_{n2}}{\sqrt{B}}\right)^2 + \ldots + \left(\frac{U_{nK}}{\sqrt{B}}\right)^2}$$

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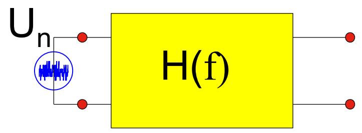
Noise transfered by linear circuits



$$U_{nO} = \sqrt{\int_0^\infty |H(f)|^2 \left(\frac{U_n(f)}{\sqrt{\Delta f}}\right)^2 df}$$

Spectral density $\left[\frac{V}{\sqrt{Hz}}\right]$

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Noise bandwidth

$$U_{nO} = \sqrt{\int_0^\infty |H(f)|^2 \left(\frac{U_n}{\sqrt{\Delta f}}\right)^2 df} \approx \sqrt{\left(\frac{U_n}{\sqrt{\Delta f}}\right)^2 B}$$

$$B = \frac{\int_0^\infty |H(f)|^2 df}{|H(f)|^2_{\max}}$$

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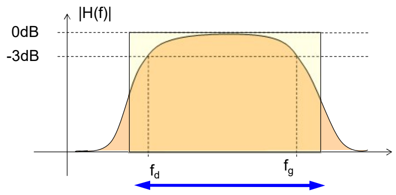
Noise bandwidth



$$1{,}57\left(f_g - f_d\right) = B > \left(f_g - f_d\right)$$

1,57 – 1,02 depends on slops
1,57 – for 20dB/dec (1rst order filter)

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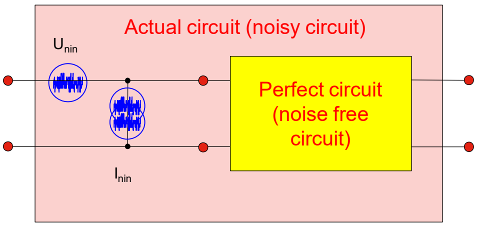
Equivalent noise sources



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OpAmp noise

Gain Bandwidth Product	GBP		2	MHz
NOISE PERFORMANCE				
Voltage Noise	e_n p-p	0.1 Hz to 10 Hz	0.5	µV p-p
Voltage Noise Density	e_n	f = 1 kHz	22	nV/√Hz
Current Noise Density	i_n	f = 10 Hz	5	fA/√Hz

	factor	C_L = 100 pF,	See Figure 1	20%	20%	
V_n	Equivalent input noise voltage	R_S = 20 Ω	f = 1 kHz	18	18	nV/√Hz
			f = 10 Hz to 10 kHz	4	4	µV
I_n	Equivalent input noise current	R_S = 20 Ω,	f = 1 kHz	0.01	0.01	pA/√Hz
		V_{rms} = 6 V	A_{VD} = 1			

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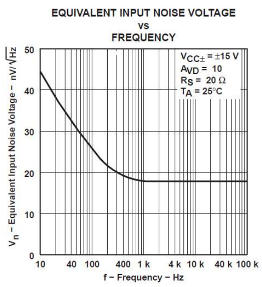

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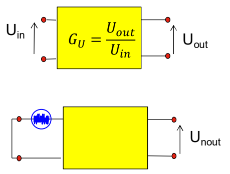
Equivalent noise sources - U_n measurement



$$U_{nin} = \frac{U_{nout}}{G_U}$$

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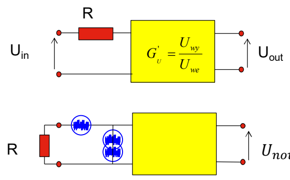
Equivalent noise sources - I_n estimation



$$U_{nout} = G'_U\sqrt{4kTRB + U_{nin}^2 + I_{ni}^2\ R^2}$$

$$I_{nin} = \frac{1}{R}\sqrt{\left(\frac{U_{nout}}{G'_U}\right)^2 - 4kTRB - U_{nin}^2}$$

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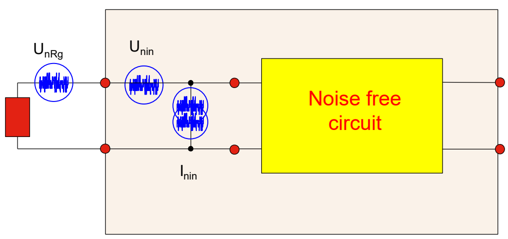
Noise factor



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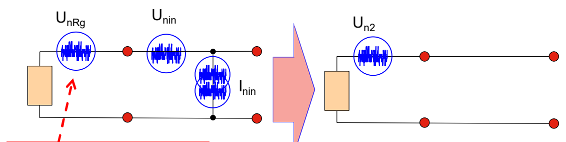
Noise factor -definition



$$U_{n1} = \sqrt{U_{nRg}^2} = \sqrt{4kTR_gB}$$

$$U_{n2} = \sqrt{U_{nRg}^2 + U_{nin}^2 + (R_g I_{nin})^2}$$

$$F = \frac{4kTR_g + U_{nin}^2 + (R_g I_{ni})^2}{4kTR_g} > 1$$

Total noise power Source + circuit

Source noise power

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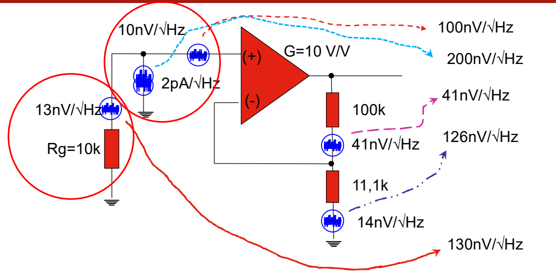
Application tips - Noise extimation



$$F = \frac{(130)^2 + (200)^2 + (100)^2 + (41)^2 + (126)^2}{(130)^2} = \frac{(130)^2 + (260)^2}{(130)^2} = 1 + 4 = 5 = 7dB$$

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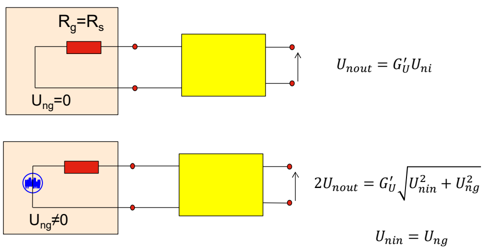
Noise factor measurement - noise generator



$$U_{nout} = G'_U U_{ni}$$

$$2U_{nout} = G'_U \sqrt{U_{nin}^2 + U_{ng}^2}$$

$$U_{nin} = U_{ng}$$

$$F = 1 + \frac{U_{ng}^2}{4kTR_g} = 1 + \frac{U_n^2 + (I_n R_g)^2}{4kTR_g}$$

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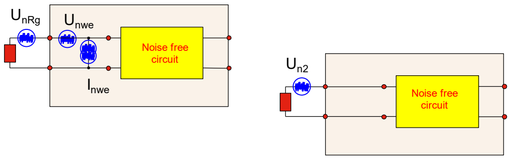
Noise factor cont.



$$U_{n2} = \sqrt{U_{nRg}^2 + U_{nin}^2 + \left(R_g I_{nin}\right)^2} = U_{nRg}\sqrt{1 + \frac{U_{nin}^2 + \left(R_g I_{ni}\ \right)^2}{U_{nRg}^2}} =$$

$$= U_{nRg}\sqrt{F} = \sqrt{4kTR_gF}$$

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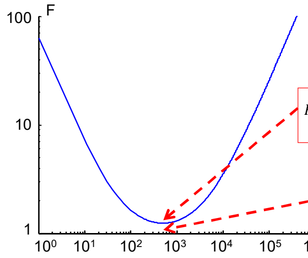
Noise matching

$$F(R_g) = \frac{4kT\ \ g + U_{nin}^2 + \left(R_g I_{nin}\right)^2}{4kTR_g}$$



$\frac{dF}{dR_g}$=0

$$F_{opt}(R_{gopt}) = 1 + \frac{U_{nin} I_{nin}}{2kT}$$

$$R_{gopt} = \frac{U_{nin}}{I_{nin}}$$

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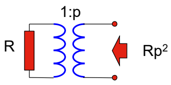
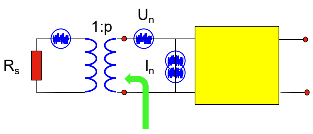
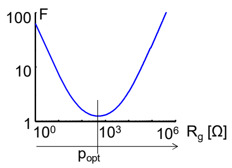
Transformer noise matching (a perfect transformer does not introduce noise)





$$F(R_g) = 1 + \frac{U_n^2 + \left(R_s p^2 I_n\right)^2}{4kTR_s p^2}$$



$$R_{gopt} = \frac{U_n}{I_n} = R_s p^2$$

$$p_{opt} = \sqrt{\frac{R_{gopt}}{R_s}} = \sqrt{\frac{U_n / I_n}{R_s}}$$

Remarcs !!!!!

- The noise matchig is NOT the power matching !!! they are differen !!!!!!
- R_{input} of circuit is not U_n/I_n !!!!
- U_n/I_n is not **resistivity** in fact - this is just a parameter !!!!

Noise factor - basic definition

$$F = \frac{4kTR_g + U_{nin}^2 + (R_g I_{nin})^2}{4kTR_g} = \frac{N_{in} + N_{circuit}}{N_{in}}$$

$$= \frac{N_{out}/G^2}{N_{in}} =$$

$$= \frac{1}{G^2}\frac{N_{out}}{N_{in}} = \frac{1}{\frac{S_{out}}{S_{in}}} \cdot \frac{N_{out}}{N_{in}} = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} =$$

$$= \frac{SNR_{in}}{SNR_{out}} > 1$$

SNR input

SNR output

Noise factor [dB]

$$F = \frac{4kTR_g + U_{nin}^2 + (R_g I_{ni})^2}{4kTR_g} \cdot \frac{G^2}{G^2}$$

Total noise power Source + circuit (on output)

Source noise power (on output)

$$F_{dB} = 10\log\left(1 + \frac{U_{ni}^2 + (R_g I_{ni})^2}{4kTR_g}\right)$$

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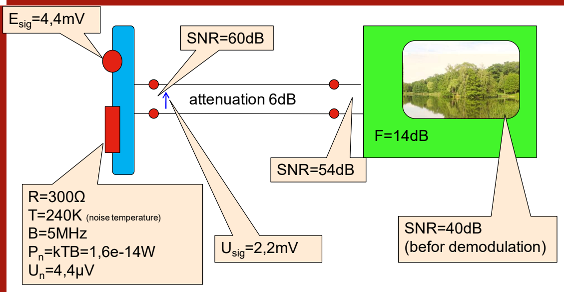
Example



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Noise temperature of an amplifier

$$F = \frac{4kTR_g + U_{nin}^2 + \left(R_g I_{nin}\right)^2}{4kTR_g} = \frac{4k(T_0 + T_n)R_g}{4kT_0R_g} = 1 + \frac{T_n}{T_0}$$

$$T_n = T_0\left(F - 1\right); \quad T_0 = 290K$$

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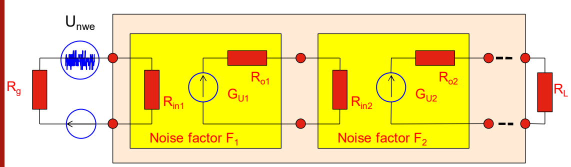
Cascade of amplifiers



$$F = F_1 + \frac{F_2 - 1}{\underbrace{\left(\frac{R_{in1}}{R_g + R_{in1}} G_{U1}\right)^2 \frac{R_g}{R_{o1}}}_{G_{P1}}} + \frac{F_3 - 1}{\underbrace{\left(\frac{R_{in1}}{R_g + R_{in1}} G_{U1}\right)^2 \frac{R_g}{R_{o1}}}_{G_{P1}} \underbrace{\left(\frac{R_{in2}}{R_{o1} + R_{in2}} G_{U2}\right)^2 \frac{R_{o1}}{R_{o2}}}_{G_{P2}}} =$$

$$= F_1 + \frac{F_2 - 1}{G_{P1}} + \frac{F_3 - 1}{G_{P1}G_{P2}} + \ldots + \frac{F_K - 1}{G_{P1}G_{P2} \cdots G_{P(K-1)}}$$

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Cascade of amplifiers cont.

$$F = F_1 + \frac{F_2 - 1}{G_{P1}} + \frac{F_3 - 1}{G_{P1}G_{P2}} + \ldots + \frac{F_K - 1}{G_{P1}G_{P2}\cdots G_{P(K-1)}} \qquad T_n = T_{n1} + \frac{T_{n2}}{G_{P1}} + \frac{T_{n3}}{G_{P1}G_{P2}} + \ldots + \frac{T_{nK}}{G_{P1}G_{P2}\cdots G_{P(K-1)}}$$

$$G_{P1}G_{P2}\cdots G_{P(K-1)} = \left(\frac{R_{in1}}{R_g + R_{in1}}G_{U1}\right)^2 \frac{R_g}{R_{o1}} \left(\frac{R_{in2}}{R_{o1} + R_{in2}}G_{U2}\right)^2 \frac{R_{o1}}{R_{o2}} \ldots =$$

$$= \underbrace{\left(\frac{R_{in1}}{R_g + R_{in1}}G_{U1}\right)^2}_{(G_{U1skuteczne})^2} \underbrace{\left(\frac{R_{in2}}{R_{o1} + R_{in2}}G_{U2}\right)^2}_{(G_{U2skuteczne})^2} \frac{R_g}{R_{o2}} \ldots =$$

$$= (G_{U1sk})^2 (G_{U2sk})^2 \cdots (G_{U(K-1)sk})^2 \frac{R_g}{R_{o(K-1)}}$$

Available Power Gain (G_a) =

=Available Output Power / Available Source Power

$$= \frac{U_{o(K-1)}^2}{4R_{o(K-1)}} \Big/ \frac{U_s^2}{4R_g}$$

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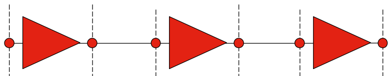
example
amplifirs + attenuators (CTV)



Noise factor	F	G	F	G	F
amplifcation	G	1/G	G	1/G	~~G~~

$$F_{total} = F + \frac{G-1}{G} + \frac{F-1}{G\cdot\frac{1}{G}} + \frac{G-1}{G\cdot\frac{1}{G}\cdot G} + \frac{F-1}{G\cdot\frac{1}{G}\cdot G\cdot\frac{1}{G}} = 3F - \frac{2}{G}$$

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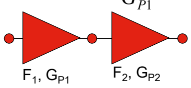
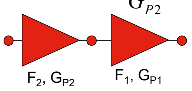
Noise factor optimization

$$F = F_1 + \frac{F_2 - 1}{G_{P1}} + \frac{F_3 - 1}{G_{P1}G_{P2}} + \ldots + \frac{F_K - 1}{G_{P1}G_{P2}\cdots G_{P(K-1)}}$$

$$F_{12} = F_1 + \frac{F_2 - 1}{G_{P1}} \quad ? \quad F_{21} = F_2 + \frac{F_1 - 1}{G_{P2}}$$



? ?



The first should be an amplifier with the lowest noise level, i.e. a low noise factor and high gain !!!!

$$\frac{F-1}{1-\frac{1}{G_P}}$$

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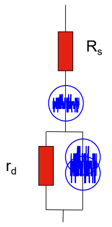
Noise of semiconductor junction



$$U_n^2 = 4kTR_S B$$

$$I_n^2 = 2qI_D B + K\frac{I_D^\alpha}{f}B$$

dynamic resistance does not hum because it is not "ohmic resistance"

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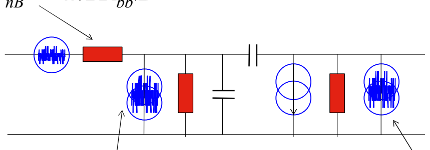
Noise of BJT

$$U_{nB}^2 = 4kTR_{bb'}B$$



$$I_{nC}^2 = 2qI_C B$$

$$I_{nB}^2 = 2qI_B B + K\frac{I_B^\alpha}{f}B$$

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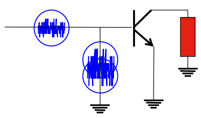
Noise of BJT

$$U_n^2 = 4kT\left(R_{bb'} + \frac{\varphi_T}{2I_C}\right)B$$

1 – 100 nV/√Hz
Low noise transistor



$$I_n^2 = 2q\left(I_B + K\frac{I_B^\alpha}{2qf} + \frac{I_C}{|\beta(j\omega)|^2}\right)B$$

0,1 – 10 pA/√Hz
Low noise transistor

$$\beta(j\omega) = \frac{\beta_0}{1 + j\frac{\omega}{\omega_\beta}}$$

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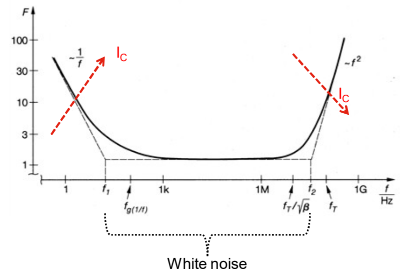
Noise factor for transistor



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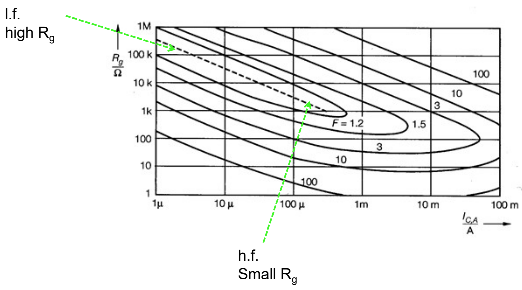
Transistor noise factor for R_g i I_C



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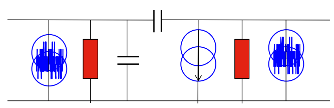
Noise of FET



$$I_{nG}^2 = 2qI_G B$$

$$I_{nD}^2 = \underbrace{4kT\left(\frac{2}{3}g_m\right)B}_{\text{szum termiczny}} + K_1\frac{I_D^2}{f}B$$

szum termiczny

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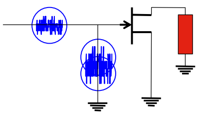
Noise of FET

$$U_n^2 = 4kT\left(\frac{2}{3g_m}\right)B + K_f\frac{B}{f}$$

1 – 1000 nV/√Hz
For low noise transistors



$$I_n^2 = 2qI_GB + \left(\frac{\omega C_{gs}}{g_m}\right)^2\left(4kT\frac{2g_m}{3} + K_1\frac{I_D^2}{f}\right)B$$

0,1 – 100 fA/√Hz
For low noise transistors

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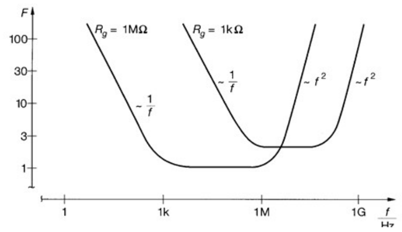
Noise of MOSFET



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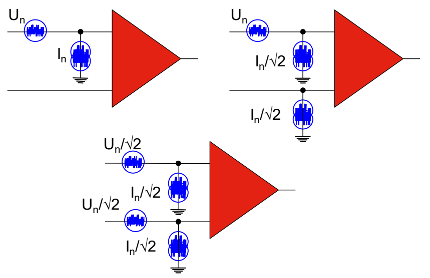
Choosing transistor

- BJT
  - is better for low R_g - source resistances (for high frequencies) - low $R_{bb'}$ and high current I_C are recommended (U_n/I_n low);
  - for high R_g the I_C should be reduced (for low frequencies);
- FET
  - is better for high source resistances ($U_n/I_n = R_{gopt}$ is high);
  - noise 1/f is higher (corner is higher);
  - low noise transistors are low frequency;

OpAmp noise



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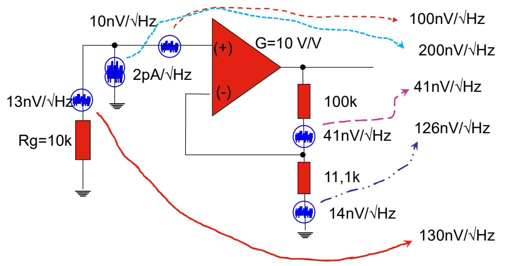
OpAmp noise
-esstimation example



$$F = \frac{(130)^2 + (200)^2 + (100)^2 + (41)^2 + (126)^2}{(130)^2} = \frac{(130)^2 + (260)^2}{(130)^2} = 1 + 4 = 5 = 7dB$$

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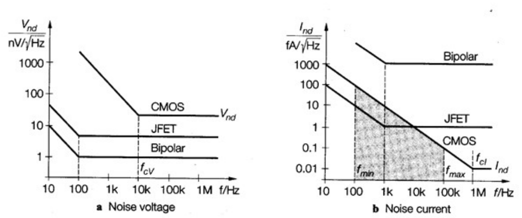
OpAmp noise - comparison



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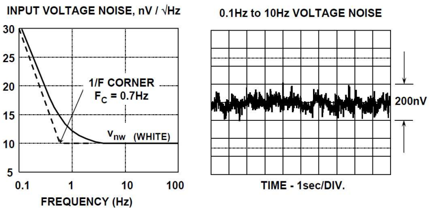

- $V_{n,rms}(F_L, F_H) = v_{nw}\sqrt{F_C \ln\left[\frac{F_C}{F_L}\right] + (F_H - F_C)}$
- For F_L = 0.1Hz, F_H = 10Hz, v_{nw} = 10nV/√Hz, F_C = 0.7Hz:
  - $V_{n,rms}$ = 33nV
  - $V_{n,pp}$ = 6.6 × 33nV = 218nV

21 ANALOG DEVICES

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Summary

- What is noise ?
- Sources (types) of noise,
- Basic relationships for thermal, shot, and flicker (1/f) noise,
- Noise bandwidth,
- Equivalent input noise sources,
- Noise factor - definition,
- Noise matching,
- Noise temperature,
- Amplifier cascade noise (noise optimization of system)
- Operational amplifier noise (comparison),
- Noise coefficient of bipolar and field-effect transistors (comparison),

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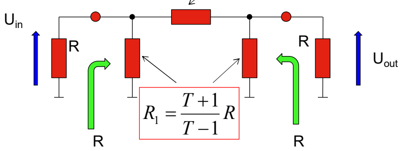
Voltage attenuator matched at both sides

Attenuation: $T = \frac{U_{in}}{U_{out}}$



$$R_2 = \frac{T^2 - 1}{2T}R$$

$$R_1 = \frac{T+1}{T-1}R$$