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Active filters

Active filters

Electronic filters are circuits with signal processing functions,

they remove unwanted frequency to enhance wanted ones.

Electronic filters can be:

- passive or active (RLC (SAW -surface acoustic wave) OR with active elements e.g. OpAmp)
- analog or digital (Signal Processing) or switched capacitor

Active filters

Types of active filters:

Due to the way filters operate, the active filters can be classified as :

- continuous time filters
- switched capacitor filters

Frequency specification:

- high-pass,
- low-pass,
- band-pass,
- band-stop (band-rejection; notch),
- all-pass (usually specific phase transmission)

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Active filters - shape of transfer function

- Czebyszev (I i II kind)
- Eliptic (Cauer)
- Bessel (Thompson)
- Butterworth - Thompson

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Active filters realization

Active filters can be realized as:

- continuous time filters – analog filters
- switched capacitor filters

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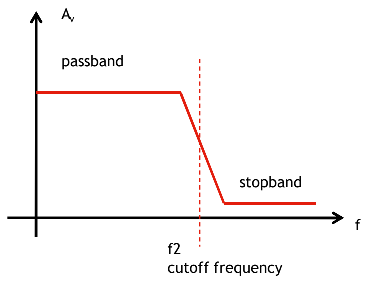
Low-Pass filter



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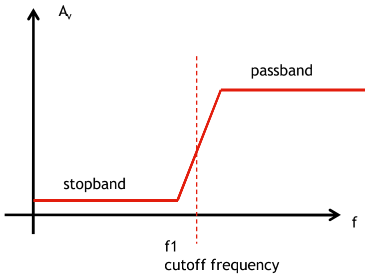
High-Pass filter



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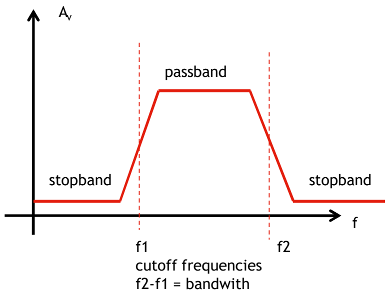
Band-Pass filter



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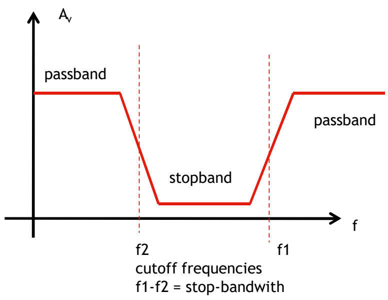
Band-Stop filter



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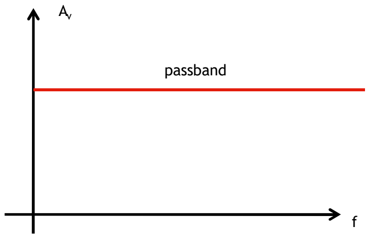
All-Pass filter (phase depends on frequency)



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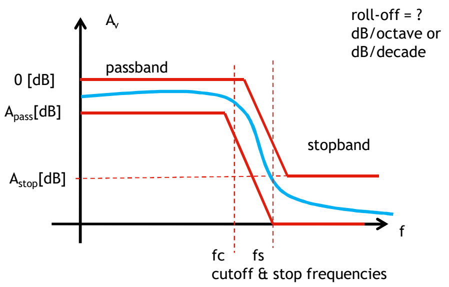
Passband and Stopband attenuation



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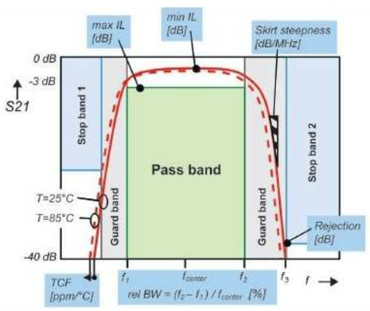

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Order of a filter

$$H(s)=\frac{U_{out}(s)}{U_{in}(s)}=\frac{a_m s^m + a_{m-1}s^{m-1}+...+a_0}{b_n s^n + b_{n-1}s^{n-1}+...+b_0}=\frac{a_m}{b_n}\frac{\prod_i (s-z_i)}{\prod_j (s-p_j)}$$

n = number of capacitors and inductances (LC - filter)
n = number of capacitors (RC - filters)

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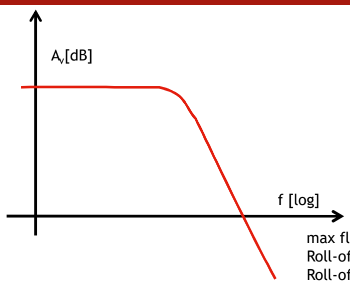
Butterworth approximation pass-band max flatness



max flatness of pass-band
Roll-off = 20*n dB/decade
Roll-off = 6*n dB/octave
-for low & high -pass
-½ for stop & pass -band
Phase/impule response = acceptable

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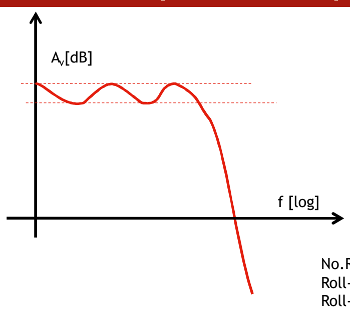
Chebyshev (I) approximation pass-band rippels



No.Rippels = n/2
Roll-off > 20n dB/decade
Roll-off > 6n dB/octave
Phase/impule response = bad

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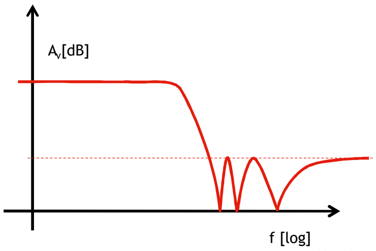
Invers Chebyshev (II) approximation stpop-band rippels



No.Rippels = n/2
Roll-off > 20n dB/decade
Roll-off > 6n dB/octave
Phase/impule response = bad

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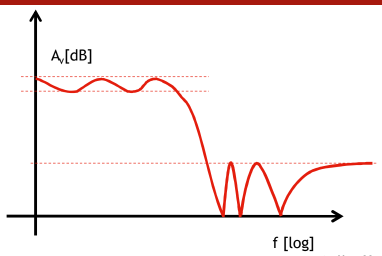
Eliptic (Cauer) approximation stpop & pass -band rippels



Roll-off > 20n dB/decade
Roll-off > 6n dB/octave
Phase/impule response = bad

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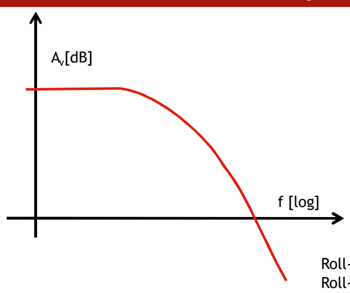
Bessel approximation ---linera phase



Roll-off < 20n dB/decade
Roll-off < 6n dB/octave
Phase/impule response = v.good
Time delay = flat v. frequency

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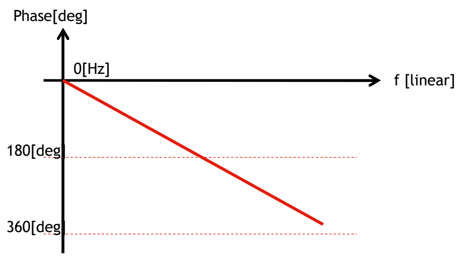
Bessel approximation ---linera phase



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6th order filter attenuation

Type	fc [dB]	2fc [dB]
Bessel	3	14
Butteworth	3	36
Chebyshev	3	63
Invers Chebyshev (II)	3	63
elliptic	3	93

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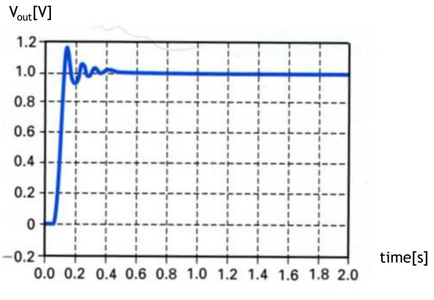
Step response Buttewoth, inv.Chebyshev (I)



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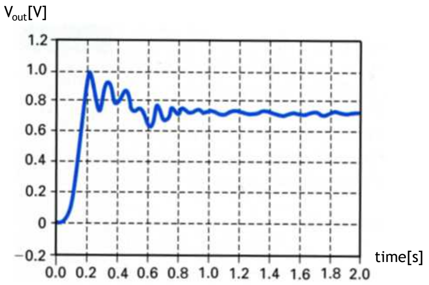
Step response
Chebyshev (II), elliptic



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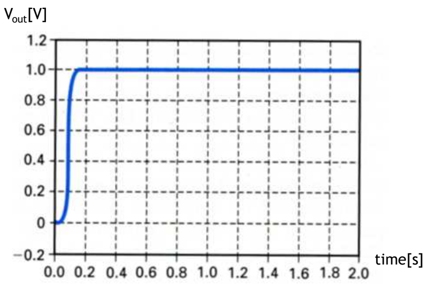
Step response
Bessel



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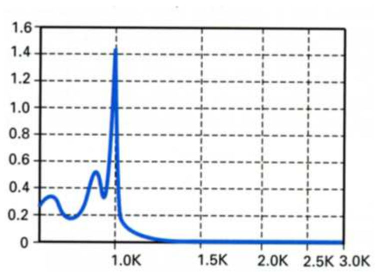
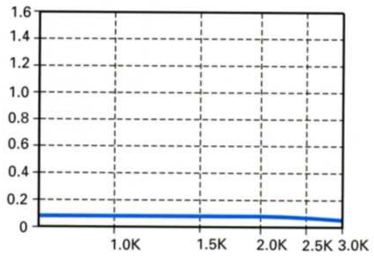
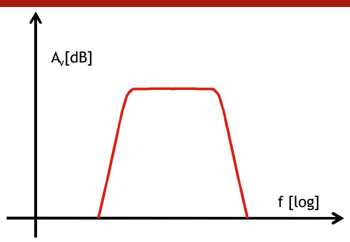
Time delay
elliptic



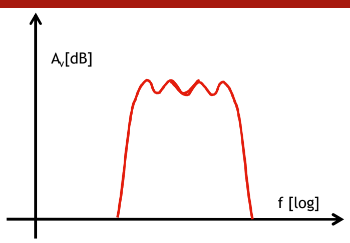
Time delay Bessel



band- pass Butteworth



band- pass Chebyshev



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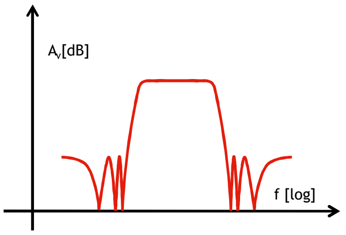
band- pass inver Chebyshev (II)



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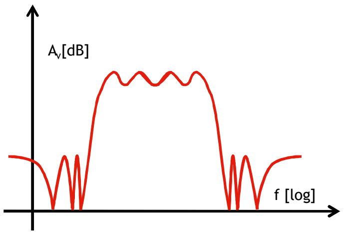
band- pass elliptic



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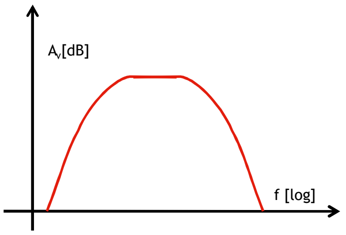
band- pass Bessel



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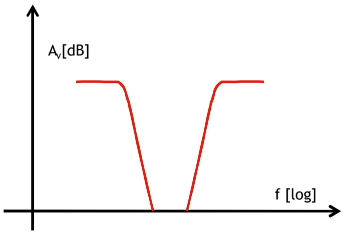
band- stop Butteworth



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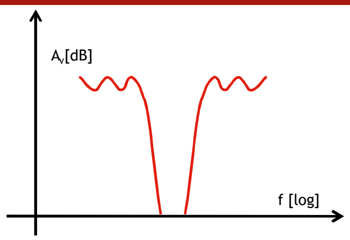
band- pass Chebyshev



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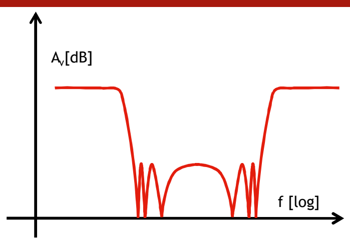
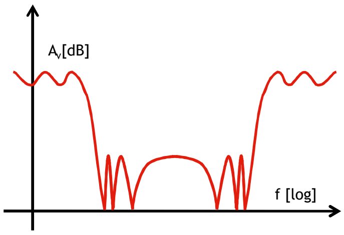
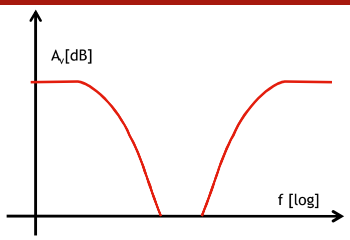
band- pass inver Chebyshev (II)



band- pass elliptic



band- pass Bessel



Filter approximation general features

Type	passband	stopband	Roll-off	Step response
Bessel	flat	monotonic	poor	best
Butteworth	flat	monotonic	good	good
Chebyshev	rippled	monotonic	v. good	poor
In.Chebyshev (II)	flat	rippled	best	poor
Elliptic (Cauer)	rippled	rippled	best	poor

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Active filter implementation

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Active filters - introduction

Transmittance:

$$H(s)=\frac{U_{out}(s)}{U_{in}(s)}=\frac{a_m s^m+a_{m-1}s^{m-1}+\ldots+a_0}{b_n s^n+b_{n-1}s^{n-1}+\ldots+b_0}=\frac{a_m}{b_n}\frac{\prod_i (s-z_i)}{\prod_j (s-p_j)}$$

a_i, b_j - real numbers

z_i, p_j - zeros and poles of transmittance

$$H(s)=|H(\omega)|\exp[j\varphi(\omega)]$$

|H(s)| - amplitude transmittance

φ(s) - phase transmittance

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Active filters - introduction c.d.

Transmittance factorization:

$$H(s)=\frac{N(s)}{D(s)}=\prod_k H_k(s)=\prod_k \frac{N_k(s)}{M_k(s)}$$

Degree of $N_k(s)$ ≤ degree of $M_k(s)$ ≤ 2

For $M_k(s)$ = 2 biquadratic section:

$$H_k(s)=\frac{a_2 s^2+a_1 s+a_0}{s^2+b_1 s+b_0}=\frac{N_k(s)}{s^2+b_1 s+b_0}$$

Active filters - intro

Biquadratic section can be expressed as:

$$H_k(s) = \frac{N_k(s)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} = \frac{N_k(s)}{(s - p_1)(s - p_2)}$$

where: p_1, p_2 - poles of the transmittance

For quality Q>1/2 poles are complex numbers

$$p_{1,2} = -\frac{\omega_0}{2Q} + j\frac{\omega_0}{2Q}\sqrt{4Q^2 - 1} = \sigma_p \pm j\omega_p$$

Active filters - intro

Real and imaginary parts are:

$$\sigma_p = -\frac{\omega_0}{2Q} \qquad \omega_p = \omega_0\sqrt{1 - \frac{1}{4Q^2}}$$

Specific frequency and quality are

$$\omega_0 = \sqrt{\sigma_p^2 + \omega_p^2} \qquad Q = \frac{\sqrt{\sigma_p^2 + \omega_p^2}}{2|\sigma_p|} = \frac{\omega_0}{2|\sigma_p|}$$

Dumping factor:

$$\alpha = \frac{1}{2Q}$$

Active filters - intro

When denominator is writen in he form:

$$s^2 + b_1 s + b_0 = s^2 + \frac{\omega_0}{Q}s + \omega_0$$

Parameters Q (quality) and ω_0 (characteristic frequency)

$$Q = \frac{\sqrt{b_0}}{b_1}$$

$$\omega_0 = \sqrt{b_0}$$

b_0, b_1 - real coefficients

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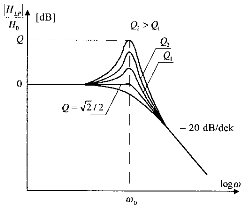
Interpretation of Q and ω_0



$$H_{LP}(s) = H_0 \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

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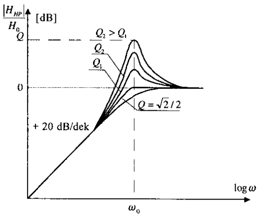
Interpretation of Q and ω_0



$$H_{HP}(s) = H_0 \frac{s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

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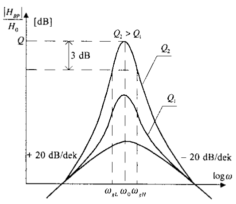
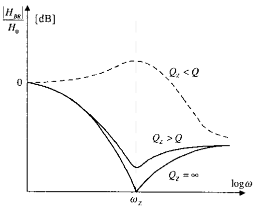
Interpretation of Q and ω_0



$$H_{BP}(s) = H_0 \frac{\omega_0 s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{\omega_{gH} - \omega_{gL}}$$

Interpretation of Q and ω_0



$$H_{BR}(s) = H_0 \frac{s^2 + \frac{\omega_z}{Q_z} s + \omega_z^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

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Filtry aktywne - wprowadzenie

$Q_z = \infty$ - eliptic fiilter with transfer function:

$$H_{BR}(s) = H_0 \frac{\omega_z^2}{\omega_0^2} \frac{\frac{s^2}{\omega_z^2} + 1}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1}$$

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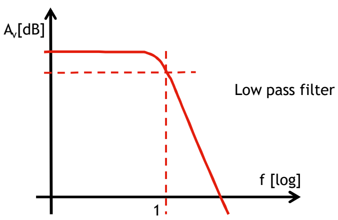
Filters transformation

$$S = \frac{s}{\omega_0} \qquad H_i(S) = \frac{Ku}{A_i S^2 + B_i S + 1}$$



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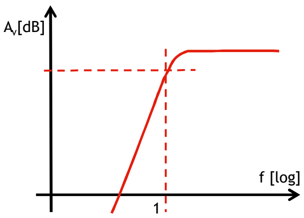
LOW → HIGH pass

$$S \to \frac{1}{S}$$

High pass filter



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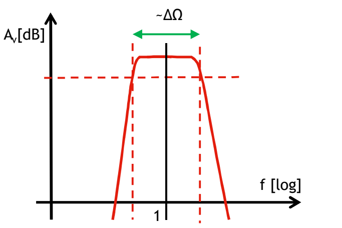
LOW → BAND PASS

$$S \to \frac{1}{\Delta\Omega}\left(S + \frac{1}{S}\right)$$

Band pass filter



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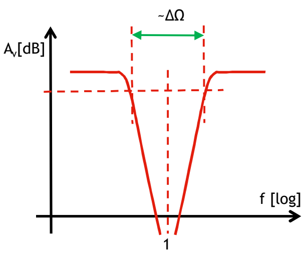
LOW → BAND STOP

$$S \to \frac{\Delta\Omega}{\left(S + \frac{1}{S}\right)}$$



Active filters - factorization

Transmittance factorization:

$$H(s)=\frac{N(s)}{D(s)}=\prod_k H_k(s)=\prod_k \frac{N_k(s)}{M_k(s)}$$

order of $N_k(s)$ ≤ order of $M_k(s)$ ≤ 2

For order $M_k(s)$ = 2 biquadratic section, or =1 first order section:

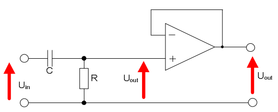
$$H_k(s)=\frac{a_2 s^2+a_1 s+a_0}{s^2+b_1 s+b_0}=\frac{N_k(s)}{s^2+b_1 s+b_0}==\frac{N_k(s)}{a_1 s^2+b_1 s+1}$$

OR

$$H_k(s)=\frac{a_1 s+a_0}{b_1 s+b_0}$$

N	i	a_i	b_i	f_{ci}/f_c	Q_i
Chebyshev filters, 0.5 dB *ripple*					
1	1	1.0000	0.0000	1.000	–
2	1	1.3614	1.3827	1.000	0.86
3	1	1.8636	0.0000	0.537	–
	2	0.6402	1.1931	1.335	1.71
4	1	2.6282	3.4341	0.538	0.71
	2	0.3648	1.1509	1.419	2.94
5	1	2.9235	0.0000	0.342	–
	2	1.3025	2.3534	0.881	1.18
	3	0.2290	1.0833	1.480	4.54
6	1	3.8645	6.9797	0.366	0.68
	2	0.7528	1.8573	1.078	1.81
	3	0.1589	1.0711	1.495	6.51
7	1	4.0211	0.0000	0.249	–
	2	1.8729	4.1795	0.645	1.09
	3	0.4861	1.5676	1.208	2.58
	4	0.1156	1.0443	1.517	8.84
8	1	5.1117	11.9607	0.276	0.68
	2	1.0639	2.9365	0.844	1.61
	3	0.3439	1.4206	1.284	3.47
	4	0.0885	1.0407	1.521	11.53
9	1	5.1318	0.0000	0.195	–
	2	2.4283	6.6307	0.506	1.06
	3	0.6839	2.2908	0.989	2.21
	4	0.2559	1.3133	1.344	4.48
	5	0.0695	1.0272	1.532	14.58
10	1	6.3648	18.3695	0.222	0.67
	2	1.3582	4.3453	0.689	1.53
	3	0.4822	1.9440	1.091	2.89
	4	0.1994	1.2520	1.381	5.61
	5	0.0563	1.0263	1.533	17.99

1st order high pass filer

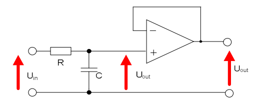


$$A_V=\frac{U_{out}}{U_{in}}=\frac{R}{R+1/sC}=\frac{sCR}{sCR+1}=$$

$$=\frac{R}{R+1/j\omega C}=\frac{1}{1+1/j\omega CR}=\frac{1}{1+\frac{1/RC}{j\omega}}=\frac{1}{1+\omega_1/j\omega}=\frac{1}{1+f_1/jf}$$

$$f_1=\frac{1}{2\pi RC}$$

1st order low pass filer



$$A_V = \frac{U_{out}}{U_{in}} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + jsCR} =$$

$$= \frac{1}{1 + j\omega \frac{1}{1/RC}} = \frac{1}{1 + j\omega/\omega_2} = \frac{1}{1 + j\, f/f_2} =$$

$$f_2 = \frac{1}{2\pi RC}$$

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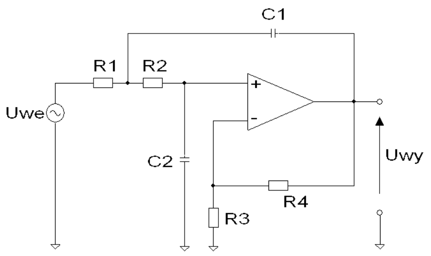
Active filters
Sallen - Key low pass implementation



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Sallen - Key low pass implementation

Transmittance:

$$H_{LP}(s) = \frac{U_{wy}(s)}{U_{we}(s)} = \frac{\dfrac{1 + R_4/R_3}{R_1R_2C_1C_2}}{s^2 + \left(\dfrac{1}{R_1C_1} + \dfrac{1}{R_2C_1} + \dfrac{R_4/R_3}{R_2C_2}\right)s + \dfrac{1}{R_1R_2C_1C_2}}$$

where:

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} \qquad Q = \frac{\dfrac{1}{\sqrt{R_1R_2C_1C_2}}}{\left(\dfrac{1}{R_1C_1} + \dfrac{1}{R_2C_1} + \dfrac{R_4/R_3}{R_2C_2}\right)}$$

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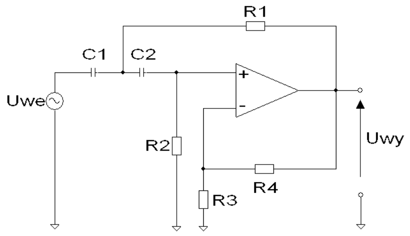
Sallen - Key high pass implementation



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Sallen - Key high pass implementation

$$H_{HP}(s)=\frac{U_{out}(s)}{U_{in}(s)}=\frac{(1+R_4/R_3)s^2}{s^2+\left(\frac{1}{R_2C_1}+\frac{1}{R_2C_2}+\frac{R_4/R_3}{R_1C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

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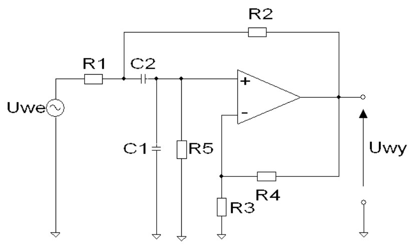
Sallen - Key band pass implementation



Sallen - Key band pass implementation

$$H_{BP}(s)=\frac{U_{out}(s)}{U_{in}(s)}=\frac{\frac{1+R_4/R_3}{R_1C_1}s}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_2}+\frac{1}{R_3C_1}+\frac{R_4/R_3}{R_2C_1}\right)s+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

$$\omega_0=\sqrt{\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}\qquad Q=\frac{\sqrt{\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}}{\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_2}+\frac{1}{R_3C_1}+\frac{R_4/R_3}{R_2C_1}\right)}$$

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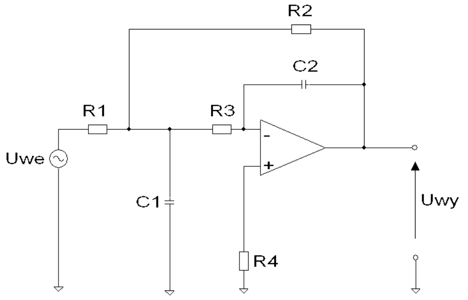
LOW PASS multiple negative feedback



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LOW PASS multiple negative feedback

$$H_{LP}(s)=\frac{U_{wy}(s)}{U_{we}(s)}=\frac{-\frac{R_2/R_1}{R_2R_3C_1C_2}}{s^2+\frac{s}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)+\frac{1}{R_2R_3C_1C_2}}$$

$$\omega_0=\frac{1}{\sqrt{R_2R_3C_1C_2}}\qquad Q=\left[\sqrt{\frac{C_2}{C_1}}\left(\sqrt{\frac{R_3}{R_2}}+\sqrt{\frac{R_2}{R_3}}+\frac{\sqrt{R_2R_3}}{R_1}\right)\right]^{-1}$$

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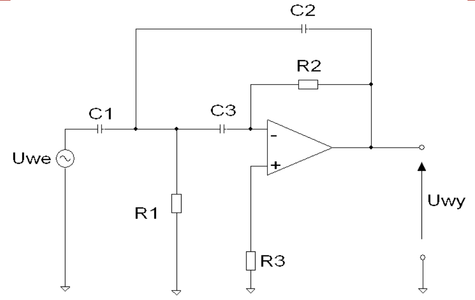
HIGH PASS
multiple negative feedback



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HIGH PASS
multiple negative feedback

Transfer function:

$$H_{HP}(s)=\frac{U_{out}(s)}{U_{in}(s)}=\frac{-C_1/C_2\,s^2}{s^2+\frac{s}{R_2}\left(\frac{C_1}{C_2C_3}+\frac{1}{C_2}+\frac{1}{C_3}\right)+\frac{1}{R_1R_2C_2C_3}}$$

where:

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_2C_3}}\qquad Q=\left[\sqrt{\frac{R_1}{R_2}}\left(\frac{C_1}{\sqrt{C_2C_3}}+\sqrt{\frac{C_3}{C_2}}+\sqrt{\frac{C_2}{C_3}}\right)\right]^{-1}$$

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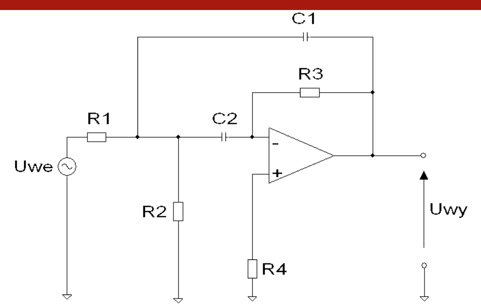
BAND PASS
multiple negative feedback



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BAND PASS
multiple negative feedback

$$H_{BP}(s)=\frac{U_{out}(s)}{U_{in}(s)}=\frac{-\frac{s}{R_1C_1}}{s^2+s\left(\frac{1}{R_3C_1}+\frac{1}{R_3C_2}\right)+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

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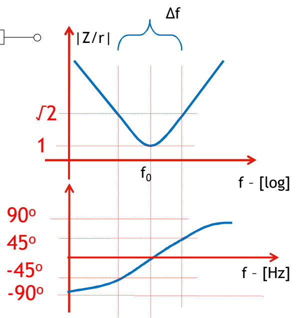
LC
resonant circuit (serial)

$$f_0=\frac{1}{2\pi\sqrt{LC}}$$

$$Q=\frac{f_0}{\Delta f}=\frac{\omega_0 L}{r}$$



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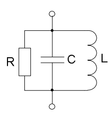
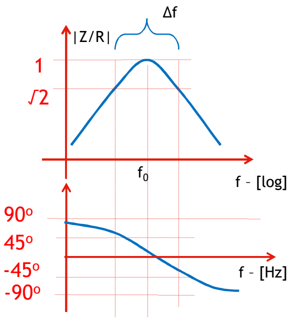
LC
resonant circuit (paralel)



$$f_0=\frac{1}{2\pi\sqrt{LC}}$$

$$Q=\frac{f_0}{\Delta f}=\frac{R}{1/\omega_0 C}$$



Q-factor defibition

$$Q = \frac{f_0}{\Delta f} = \frac{\omega_0}{\omega_{gH} - \omega_{gL}}$$

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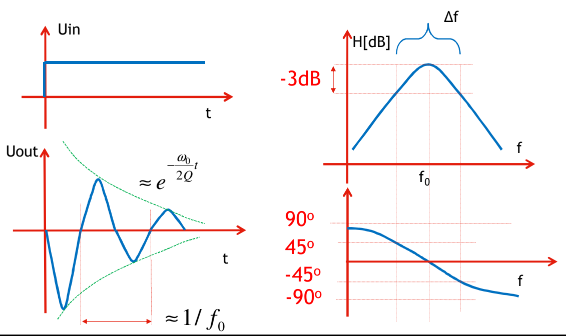
Band pass



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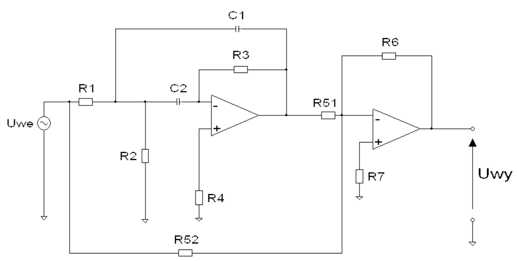
BAND STOP



Filtry aktywne - wielokrotne, ujemne sprzężenie zwrotne

Transmitancja filtru:

$$H_{BR}(s) = -\left(\frac{R_6}{R_{51}} H_{BP}(s) + \frac{R_6}{R_{52}}\right)$$

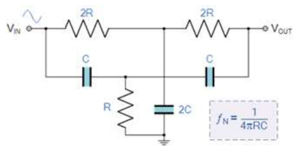
Przy częstotliwości środkowej $\omega = \omega_0$

$$\left.\begin{array}{l} |H_{BP}(\omega = \omega_0)| = 1\frac{V}{V} \\ \varphi(\omega = \omega_0) = \pi \end{array}\right\} H_{BP} = -1\frac{V}{V}$$

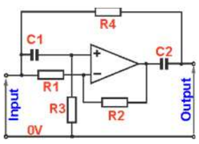
Dla: $R_{51} = R_{52} = R$

$$H_{BR}(\omega = \omega_0) = 0$$

BAND STOP (NOTCH FILTER)



$C1 = C2 = C$
$R4 = R3 = R$
$R2 = R1 = R'$
$f_0 = 1/2\pi RC$



ALL PASS FILTER

$$H(s) = \frac{N(s)}{D(s)} = \prod_k H_k(s)$$

$$H_k(s) = \frac{s^2 - a_1 s + a_0}{s^2 + b_1 s + b_0}$$

$$|H_k(s)| = 1$$

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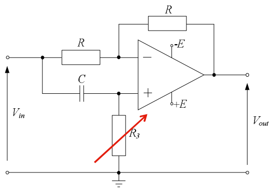
phase shifter (all pass filter of 1st order)



$$\frac{V_{out}}{V_{in}} = -\frac{1 - sCR_3}{1 + sCR_3}$$

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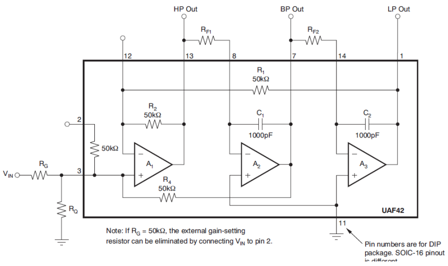
Biquadrqtic monolithic universal filter UF42 by TI (state variable filter)



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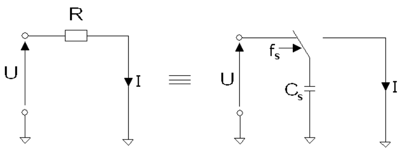
Switched capacitor filter



$$I = \frac{U}{R}$$

$$I = \frac{dQ}{dt} = \frac{d(UC_S)}{dt} = \frac{UC_S}{T_S} = UC_S f_S$$

$$R = \frac{1}{C_S f_S}$$

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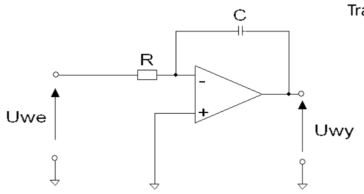
C switched filter



Transfer function

$$\frac{U_{wy}}{U_{wy}} = -\frac{1}{\tau s}$$

Where: $\tau = RC$

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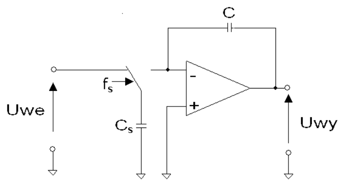
C switched filters



$$\frac{U_{wy}}{U_{wy}} = -\frac{1}{\tau s}$$

$$\tau = CR_{ZAST} = \frac{C}{f_S C_S} = \frac{\eta}{2\pi f_S}$$

$$\frac{\eta}{2\pi} = \frac{C}{C_S}$$

$$\eta = (50 \div 200)$$

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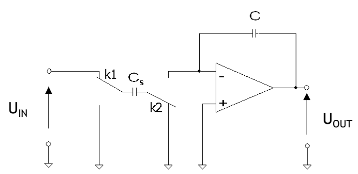
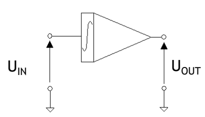
C switched integrator





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C switched integrator

$$\frac{U_{wy}}{U_{we}} = \frac{f_S C_S}{sC} = \frac{1}{\tau s}$$

$$\tau = CR_{ZAST} = \frac{C}{f_S C_S} = \frac{\eta}{2\pi f_S}$$

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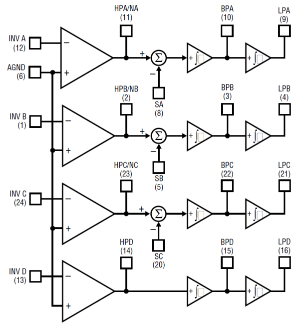
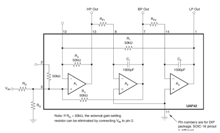
4 biquadratic filters with switched capacitors in LTC1064 by LT





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Problems

1. Q-factor of a bandpass filter ?
2. Types of filters (band stop, b. pass etc.)
3. Types of frequency response of filters (Bessel, …..)
4. Biquadratic filter (biquad section)
5. Active filters topologies (Sallen – Key, multiple feedback, state variable, C-switching)